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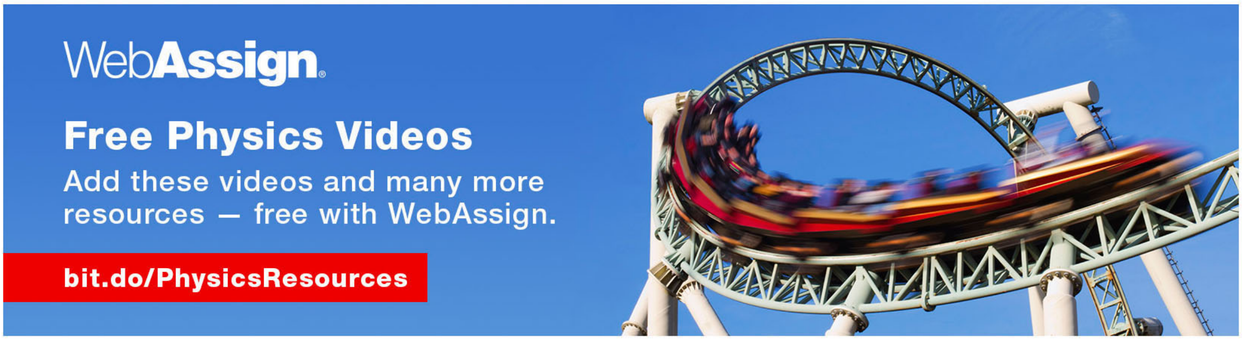

Space Curvature and the “Heavy Banana ‘Paradox’”

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The pedagogical problem of general relativity’s (GR) space curvature is twofold. First, space curvature has been difficult to visualize. The conventional diagram (Fig. 1) wherein mass causes light to bend into a “gravitational well”[1] correctly indicates that an observer can see the curvature of a light path. However, it can be misleading in that no observer can actually see the curvature of space. Moreover, space and light curvature are not synonymous. Second, aside from some outstanding reviews of various aspects of GR[2-4] satisfactory qualitative proofs[5] as to the validity of space curvature are few. We attempt here to solve both problems.

Visualization of the concept of space curvature is greatly enhanced by viewing it as meterstick contraction;[6] that is, mass can be thought of as causing metersticks (and in fact all matter) to contract. The qualitative proof of that requires only a small knowledge of special relativity (SR) and is based on the principle of equivalence (POE).

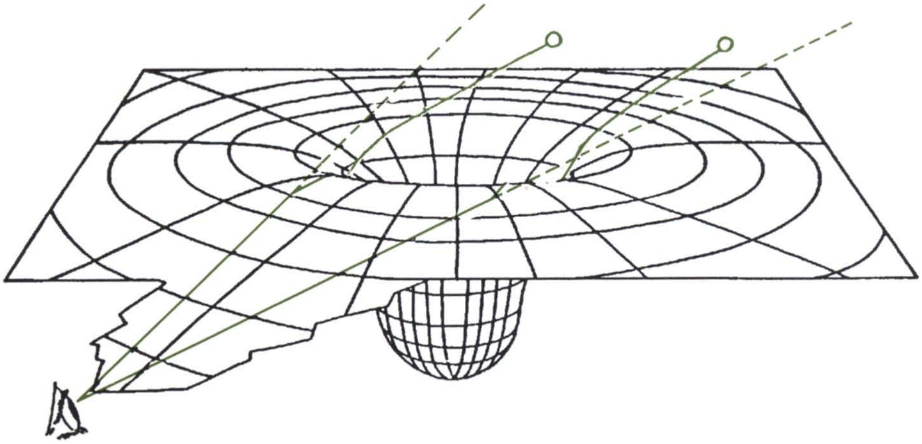

Fig. 1. The “gravitational well” of general relativity can be misleading in that no observer can see the curvature of space, only the curvature of light.

The POE states that in any small region of space the effects produced by gravitation are the same as those produced by an acceleration.[7] A good example is that of two observers, one in a rocket ship (O_r) and the other in a small room on the surface of a large planet (O_p), neither of whom can see out. Both experience a downward force and shine a light perpendicular to the direction of the perceived force. O_r will note the photons to take a curved path toward the floor. According to the POE, O_p should note the same trajectory. Consequently, the observers cannot tell whether they are in a rocket or on the surface of a large planet unless able to look out the window.

One of the effects of acceleration (and deceleration) is meterstick contraction. It’s similar to the familiar ladder-in-the-barn paradox, in which a boy with a 2.5-m ladder runs into a 2.0-m barn and the door shuts behind him. Something (either the door or ladder) must give when he comes to rest. According to the boy, he will never see the ladder fit into the barn. This is because

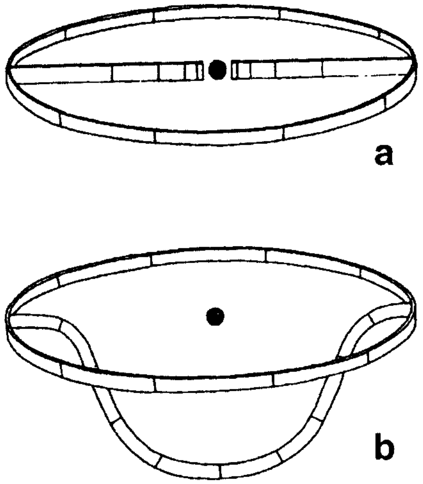

Fig. 2. a) Pedagogically, space curvature is best visualized as meterstick contraction. Keeping the ring fixed as mass is introduced, radial metersticks contract leaving room for more. Thus $C \neq \pi D$. b) The alternative view is that space expands to accommodate more metersticks of the same size.

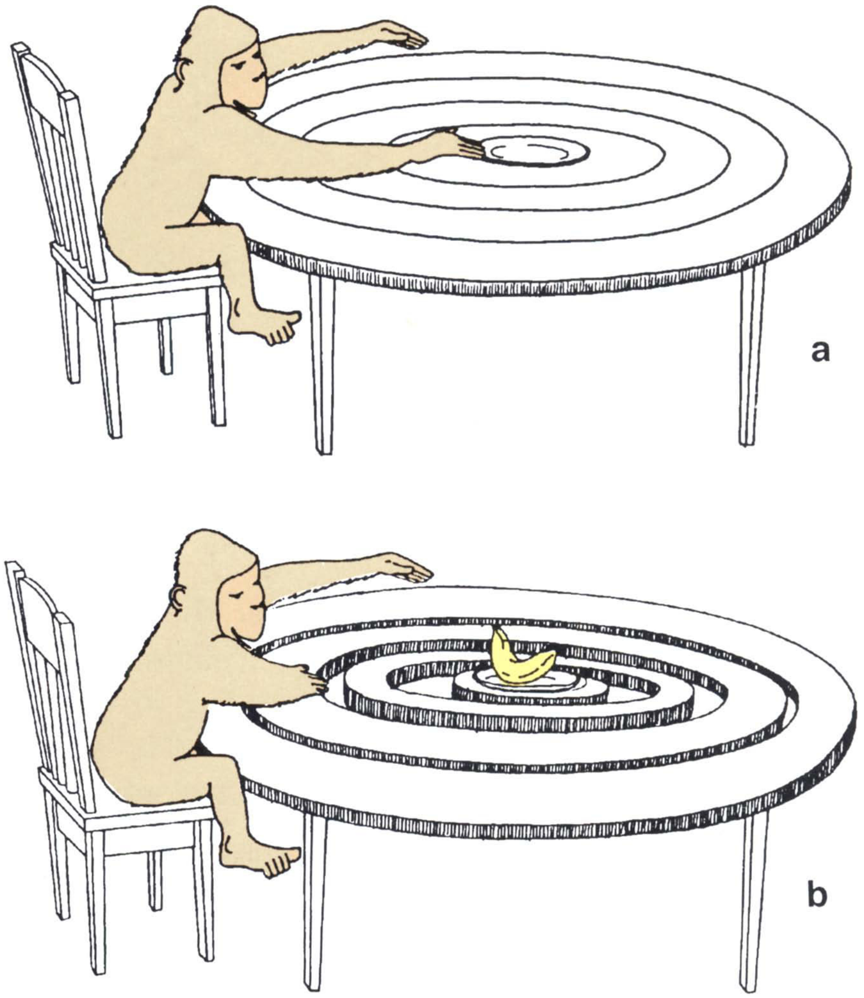

Fig. 3. The heavy banana "paradox": a) Normally, the monkey can reach the plate. b) If a massive banana is placed on the plate, the monkey's arm (and the rings that form the table) contract, preventing him from reaching the banana.

information cannot get from the front end of the ladder to the back end (where the boy is) faster than the velocity of light (c), so by the time the boy at the back end knows something is up, the front end has collided with the end of the barn. To the distant observer, the ladder does fit in the barn with the door closed.

Mathematically, this is demonstrated by considering a ladder of length L. At $t = 0$, the boy decelerates the back end with a deceleration of a. Information about his pushing the back end reaches the front end a time, $\Delta t = (L - \Delta L)/c$, later, by which time the back end has moved a distance

$$\Delta L = 0.5\, a\, (\Delta t)^2 \tag{1}$$

Is this meterstick contraction as real as the clock slowing (and lack of aging) in the twin paradox? Consider a ring and radius of metersticks (Fig. 2). Introduce enough mass to the center so that the force at the ring is still negligible. From the POE, radially placed metersticks will appear contracted to a distant observer. This would leave gaps between the metersticks. If the gaps are apparent (not real), a particle could exist between the gaps without being seen. And that is impossible. Therefore, the gaps are real in that they can be filled with more metersticks. Thus, the total number of metersticks required to span the radius is greater than it was prior to mass introduction (Fig. 2a);[7] i.e.,

$$C \neq \pi D \tag{2}$$

in a gravitational field. And so we have a need for non-Euclidean geometry, and refer to this phenomenon as "space curvature" (Fig. 2b).

The heavy banana "paradox"[8] is perhaps the most easily visualized consequence of space curvature and does for GR what the twin paradox does for SR. A monkey sitting at a table is just able to reach a plate at the table's center (Fig. 3a). If a massive banana is placed on the plate, the monkey will not be able to reach it (Fig. 3b). In fact, if the mass of the banana is increased to 10^{24} kg (the mass of the Earth), which is theoretically possible, his fingers miss the banana by approximately 1 cm. It is assumed that the hypothetical monkey and table have been reinforced so as to withstand the potentially crushing force of such a mass. What is occurring, of course, is that the radial arm (and the rings comprising the table) contract, especially those segments closest to the massive banana. The monkey simply doesn't have enough arm to reach the banana. An external observer would actually be able to see this phenomenon as described. It is referred to as a "paradox" only because of its highly non-intuitive nature.

There is a corollary to this paradox: a monkey sitting at a table and holding onto a centrally located massive banana will be able to reach further around the table with his free hand than if no banana were there. What is occurring, of course, is that the massive banana causes the radial arm to contract. The monkey and table perimeter are drawn closer to the banana, allowing the free hand to reach further around the table.

An alternative (and conventional) view is that metersticks don't need to be thought of as contracting. They can remain the same length, but only if one insists that the space dilate (or curve). Both views correctly describe the mass effect (Fig. 2).[6,7] For relativists it is mathematically more convenient to view the mass effect as space curvature. Curvature is not necessary otherwise.[9] For pedagogic reasons, however, it can be more convenient to view the mass effect as contraction.

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Physics Trick of the Month

Stabbing an Eggshell

Put half an eggshell over the end of a sharp-pointed kitchen knife. Problem: Tap the handle on a table or counter and cause the point of the knife to penetrate the shell. When others try it, the shell refuses to crack. Even after you show how it's done, your friends will still be unable to do it unless they are very observant.

Secret: Appear to tap the handle on the table, but actually hold it loosely so that when the knife strikes the table it bounces.

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