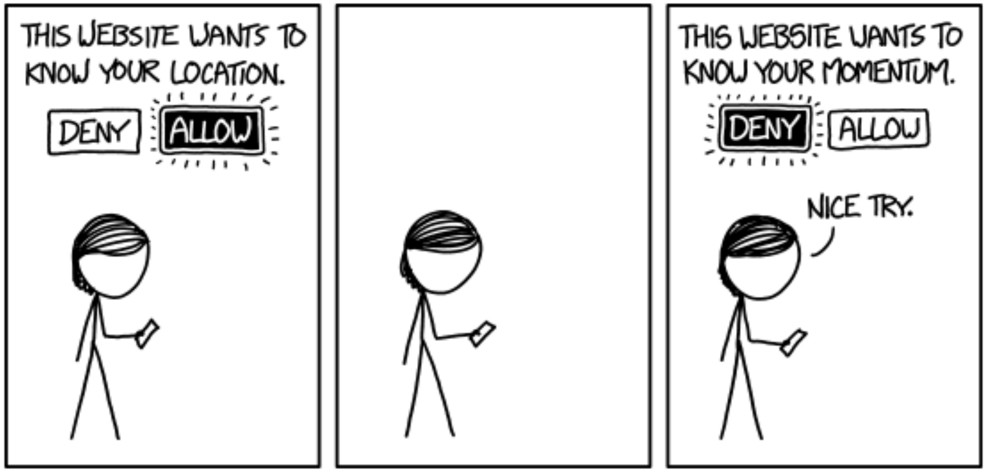
What Heisenberg Knew: A QuarkNet Data-Based Activity



xkcd

AAPT Winter Meeting 2021

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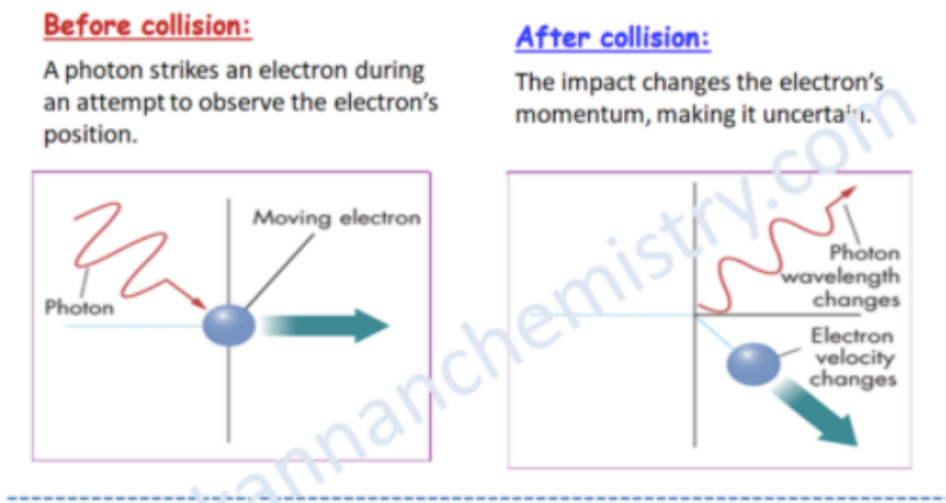
My Understanding

- **"The very act of measuring the position of the electron changes its momentum, making its momentum uncertain."—kannanchemistry.com**



Criticized as being too simplified, missing the wave nature

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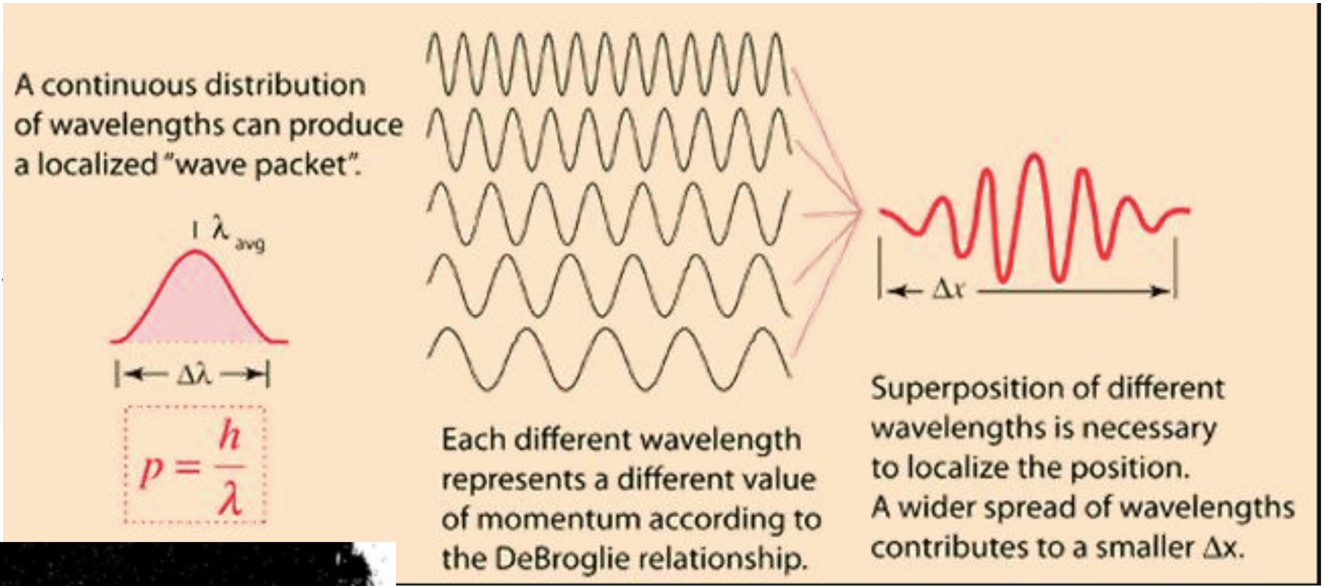
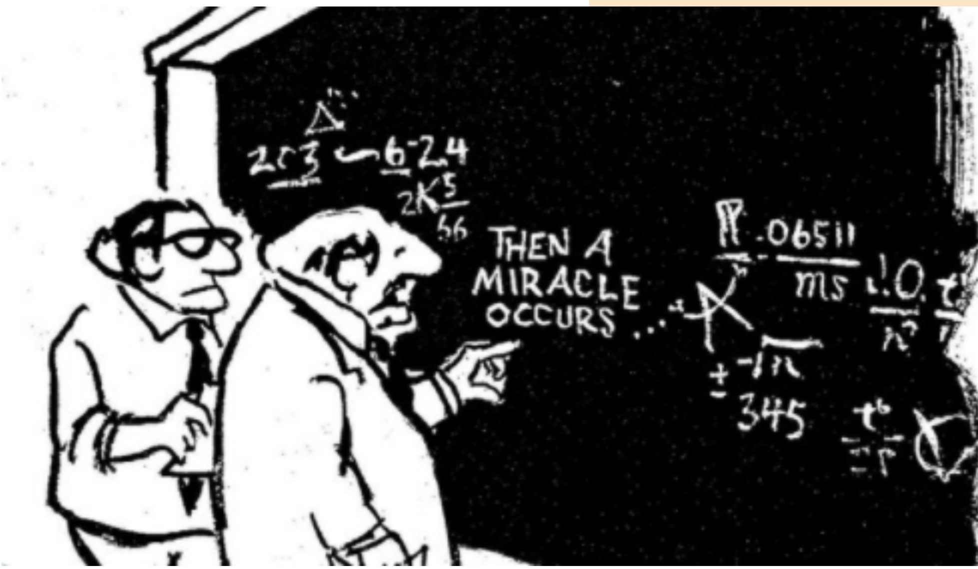
How Theorists Have Explained It

$$
\begin{aligned}
g(x) &= \frac{1}{\sqrt{2\pi\hbar}} \cdot \int_{-\infty}^{\infty} \tilde{g}(p) \cdot e^{ipx/\hbar}\, dp \\
&= \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} p \cdot \varphi(p) \cdot e^{ipx/\hbar}\, dp \\
&= \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \left[p \cdot \int_{-\infty}^{\infty} \psi(\chi) e^{-ip\chi/\hbar}\, d\chi \right] \cdot e^{ipx/\hbar}\, dp \\
&= \frac{i}{2\pi} \int_{-\infty}^{\infty} \left[\psi(\chi) e^{-ip\chi/\hbar} \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \frac{d\psi(\chi)}{d\chi} e^{-ip\chi/\hbar}\, d\chi \right] \cdot \\
&= \frac{-i}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d\psi(\chi)}{d\chi} e^{-ip\chi/\hbar}\, d\chi\, e^{ipx/\hbar}\, dp \\
&= \left(-i\hbar \frac{d}{dx} \right) \cdot \psi(x),
\end{aligned}
$$

Wikipedia



ReadingFeynman.org



Sydney Harris, 1977

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Using Data to Discover

"What Heisenberg Knew"

QuarkNet Activity in the Data Portfolio (QuarkNet.org)

- Students analyze momentum and position data to discover the uncertainty principle.
- Real data from hot fullerene molecule diffraction (Nairz, Arndt & Zeilinger, 2001)
- Let's explore!

WHAT HEISENBERG KNEW
STUDENT PAGES

DESCRIPTION

Werner Heisenberg (1901–1976) was one of the most important physicists in the creation of quantum mechanics. In 1927, he proposed the uncertainty principle. It stated that pairs of complementary variables in physics had minimal measurement uncertainties based on a relationshi with each other: less uncertainty in one inevitably yields greater uncertainty in the other, no matter how sophisticated the measurement technique. Your task is to determine if the experimental data provided represents complementary variables by testing for an inverse relationship.

What do we know?

- These data represent the uncertainty in position and momentum as determined by a diffraction experiment in 2001. See Figure 1 below.

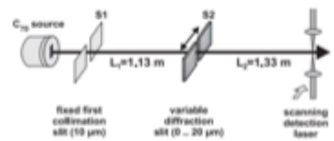


Figure 1: *Experimental setup made by Nairz, Arndt, and Zeilinger, 2001,* *https://arxiv.org/abs/quant-ph/0105061*.

- The uncertainty in position is determined using the slit width.
- The uncertainty in momentum is determined using the width of the central maximum.
- The steps for linearizing graphed data

What do we need?

- **Data Table A: Complementary Variables Momentum (p) and Position (x)**

Uncertainty in Position, Δx (micrometers)	Uncertainty in Momentum, Δp ($\times 10^{-27}$ kg-m/s)	Reciprocal 1/Δx (1/ μm)
0.09	9.6	
0.28	2.8	
0.46	1.3	
0.65	1.0	
1.36	0.5	
2.52	0.3	

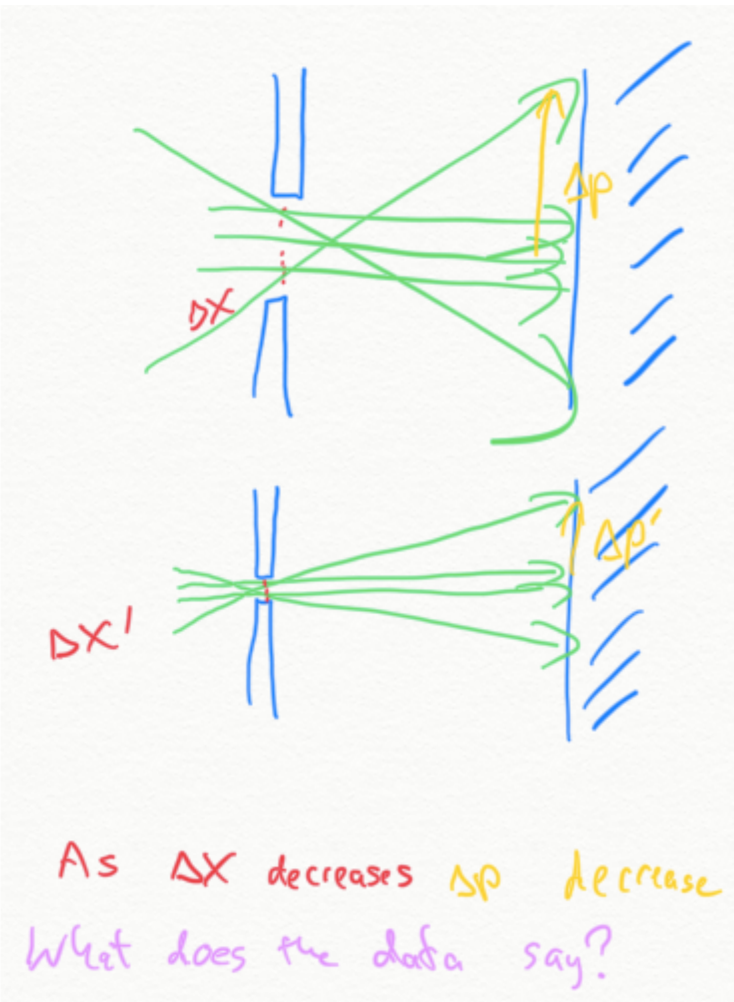
- Graph paper or a graphing program

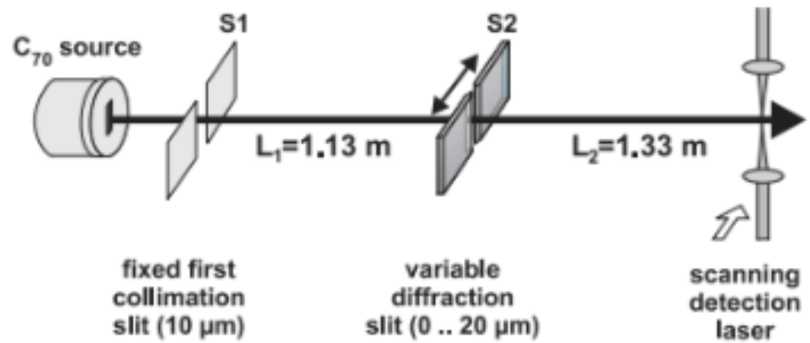
What do we do?

- **Plot** Δp vs. Δx.
- **Describe** the shape of the graph.
- **Make a claim** about what happens to Δx when Δp increases.

Expectations

How Particles Should Behave





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What the Data Says

- Data Table A: Complementary Variables Momentu

Uncertainty in Position, Δx (micrometers)	Uncertainty in Momentum, Δp ($\times 10^{-27}$ kg-m/s)
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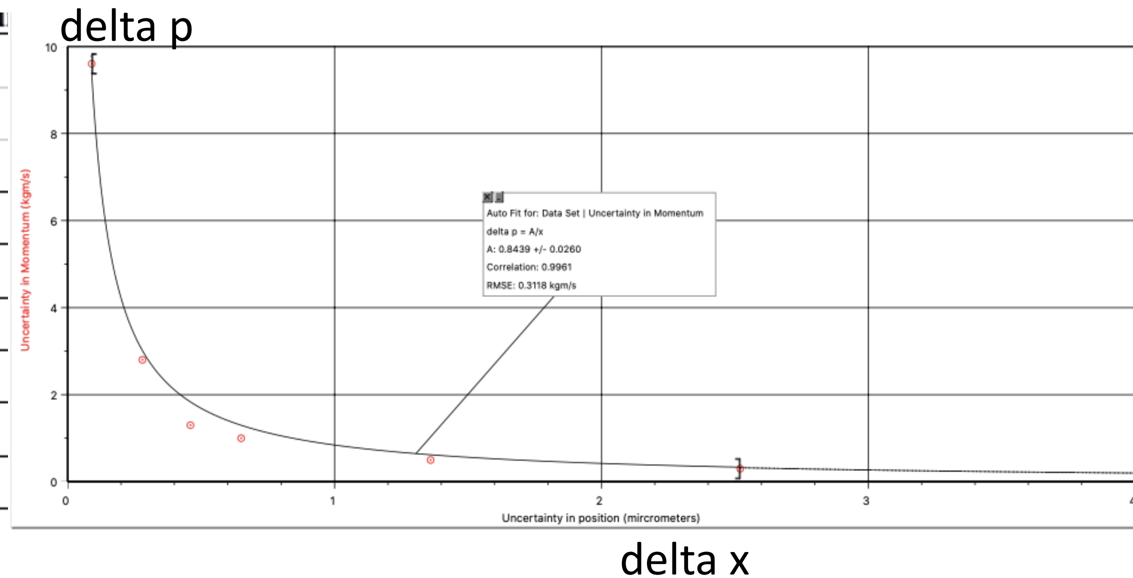

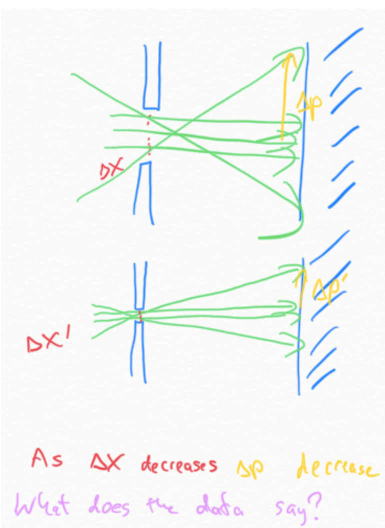

- **We observe the reverse!**

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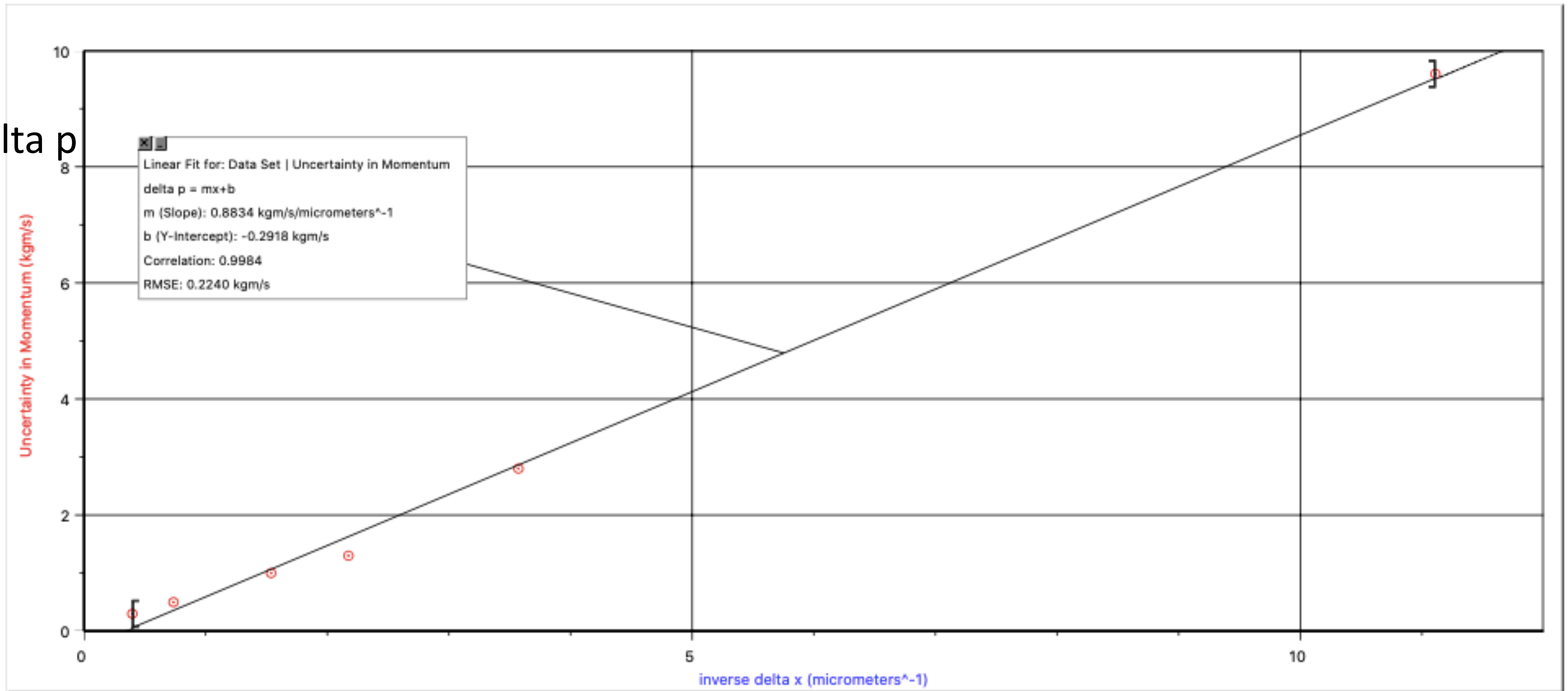
Linearize

Slope = 8.83 x 10^-34 kgm^2/s

Ball Park of Planck's Constant

delta p



1/(delta x)

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Explaining the Data

How far do you want to go with your students?

- Merely state the inverse relationship.
- Derive the Heisenberg relationship from diffraction using de Broglie (or vice versa).
- Explore particle-wave duality.

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Big Take-Aways

What do we want students to know?

- The "smear" of the central maxima refutes a particle model.
- There is an inverse relationship between the uncertainties in position and momentum.
- The relationship is not due to a collision between particles or a lack of precision in the measurement.
- Uncertainty is a fundamental property based on the wave-like nature of matter.