Z MASS SPREADSHEET EXTENSION

TEACHER NOTES

The purpose of this extension of the Z Mass activity is to add the use of a spreadsheet to support data analysis. Students use computational thinking skills to build a spreadsheet to analyze a large data set. They then build a histogram to aid in interpreting the results.

STANDARDS

Next Generation Science Standards

Science Practices

2. Developing and using models
4. Analyzing and interpreting data
5. Using mathematics and computational thinking
6. Constructing explanations
7. Engaging in argument from evidence
8. Obtaining, evaluating, and communicating information

Disciplinary Core Ideas – Physical Science

PS1.A: Structure and Properties of Matter
PS2.B: Types of Interactions
PS3.B: Conservation of Energy and Energy Transfer

Crosscutting Concepts

1. Patterns.
3. Scale, proportion, and quantity.
4. Systems and system models.

Common Core Literacy Standards

Reading

9-12.4 Determine the meaning of symbols, key terms . . .
9-12.7 Translate quantitative or technical information . . .

Common Core Mathematics Standards

MP2. Reason abstractly and quantitatively
MP6. Attend to precision.

IB Physics Topic 1: Measurement and Uncertainty

1.2.6 Describe and give examples of random and systematic errors.
1.2.8 Explain how the effects of random errors may be reduced.
1.2.11 Determine the uncertainties in results.
1.3.1 Distinguish between vector and scalar quantities.
1.3.2 Combine and resolve vectors.

IB Physics Topic 2: Mechanics

2.3.6 Use the principle of conservation of energy to compare an initial state to a final state.
2.4.3 Use conservation of linear momentum to compare an initial state to a final state.

IB Physics Topic 7: The Structure of Matter

Aim 4: particle physics involves the analysis and evaluation of very large amounts of data
Standard 7.3.4: Apply the Einstein mass-energy equivalence relationship

IB Physics Additional Higher Level Option Topic A.4: Relativistic Mechanics

A.4.6 Use MeV c^{-2} or as GeV c^{-2} the unit of mass and MeV c^{-1} or GeV c^{-1} as the unit of momentum.

A.4.7 Describe the laws of conservation of momentum and conservation of energy within special relativity

A.4.10 Solve problems involving relativistic energy and momentum conservation in collisions and particle decays

ENDURING UNDERSTANDINGS

- Scientists can analyze data more effectively when they are properly organized; charts and histograms provide methods of finding patterns in large data sets.

LEARNING OBJECTIVES

Students will be able to:

- "Code" a spreadsheet to complete calculations.
- Construct a histogram of the results.
- Make a claim based on the histogram with supporting evidence and reasoning.

PRIOR KNOWLEDGE

The students have completed the original Z mass activity in which they compute the mass of the Z boson by hand.

What do we know?

1. Momentum is conserved. Energy is too.
2. Momentum is a vector. Energy is not.
3. The invariant mass of the Z boson becomes the momentum and mass of the $\mu^+\mu^-$ pair.
4. The net momentum of the $\mu^+\mu^-$ pair is the same as the net momentum of the Z boson.
5. The muon mass is small. In these events, we can say that the muon energy (in GeV) and momentum (in GeV/c) are equivalent.
6. Einstein actually wrote $E^2 = p^2c^2 + m^2c^4$. The full derivation can be found at this link: https://quarknet.org/page/energy-momentum-and-mass
   This allows us to solve for energy, momentum or mass using $E^2 = p^2 + m^2$, where the c value is absorbed into the units.

IMPLEMENTATION

1. **Building the Technique**
   You may have done the original Z mass activity in one of several ways. This technique for finding the Z mass from a single event is designed to help the students understand how to build a spreadsheet to do the analysis. If your students do a complete calculation by hand, they then have a basis for checking each step of their spreadsheet analysis.

2. **By-Hand Calculation**
   Pick one event and have the students work collaboratively *on the same event* to calculate the Z mass **by-hand**.
   - For each muon:
     - Find the angle from the positive x axis to the direction of the muon track. [angle 1(deg), angle 2 (deg)]
     - Record the magnitude of the momentum. [p1 (GeV/c), p2 (GeV/c)]
     - Record the energy. [E1 (GeV), E2 (GeV)]

- Determine the x component of the momentum. [p1x (GeV/c) = p1 cos(θ), p2x (GeV/c) = p2 cos(θ)]
- Determine the y component of the momentum. [p1y (GeV/c) = p2 sin(θ), p2y (GeV/c) = p2 sin(θ)]

- For the event:
  - Find the net momentum in the x direction. [px(net) (GeV/c) = p1x (GeV/c) + p2x (GeV/c)]
  - Find the net momentum in the y direction. [py(net) (GeV/c) = p1y (GeV/c) + p2y (GeV/c)]
  - Find the net energy of the event [E(net) (GeV) = E1 (GeV) + E2 (GeV)]. The value of E1 (GeV) is equivalent to the magnitude of p1 (GeV/c) and the value of E2 (GeV) is equivalent to the magnitude of p2 (GeV/c). This practice works for the low mass, high momentum nature of the electrons and muons. Effectively [E(net) (GeV) = p1 (GeV) + p2 (GeV)]
  - Find the magnitude of the resultant event momentum. $p(net) = \sqrt{(px(net)^2 + py(net)^2)}$
  - Calculate the Z mass using $E^2 = p^2c^2 + m^2c^4$. The units we use for energy momentum and mass enable us to right the equation with c is absorbed into the units and cancels out. For more information follow this link: https://quarknet.org/page/energy-momentum-and-mass Therefore we will use the equation as $E^2 = p^2 + m^2$
  - The value of E1 (GeV) is equivalent to the magnitude of p1 (GeV/c) and the value of E2 (GeV) is equivalent to the magnitude of p2 (GeV/c). This practice works for the low mass, high momentum nature of these particles.

- You and your students can review how to make a histogram: https://quarknet.org/content/making-histograms

3. **Introduction to Spreadsheets**
   Students create a spreadsheet that includes all the necessary columns for the calculation of the Z mass. Encourage your students to use the column headings given above.

   NOTE: The expressions in brackets are recommended column headers in the spreadsheet.

4. **Entering Data**
   a. For each muon, enter the value of the angle, the magnitude of the momentum and the value of the energy.
   b. For calculated columns, enter the "code" for finding each value. Fill down the column.
   c. It is likely that many students will initially get a different value for the Z mass using their spreadsheet than they did in their "by-hand" calculation. Small differences are a normal occurrence; students should troubleshoot large differences to understand and correct the discrepancy. Some causes for a large discrepancy may include:
      - Errors in the "by-hand" calculation.
      - Failure to take into account that the spreadsheet defaults to angle measures in radians.
      - Incorrect formula for the calculation.
      - Incorrect entry of positive and negative signs.

      Teacher key in a Google Sheet:
      https://docs.google.com/spreadsheets/d/1CInVptCsWdnJrJDKzWtkhHH2DycTBN5GGh25xCgwo4A/copy?usp=sharing

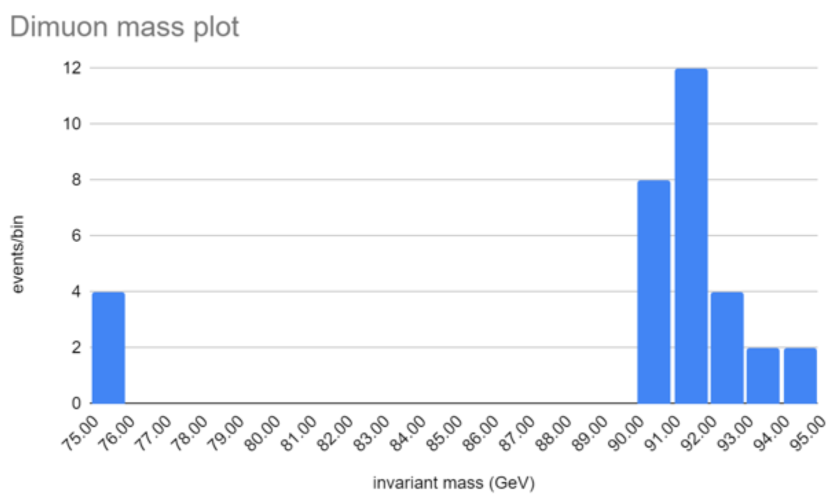
d. Once students are confident that their spreadsheet correctly calculates the mass of the Z boson, each group then enters the data into the spreadsheet from the other seven events.

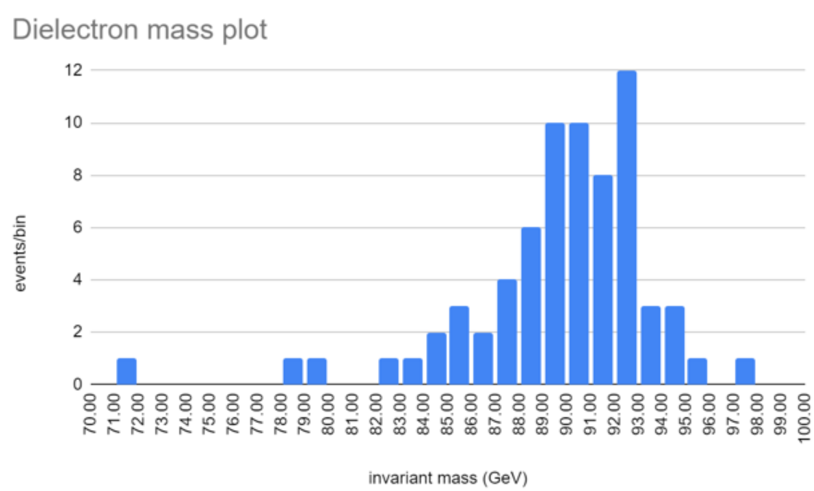
e. Students create a histogram for the eight events using tools in the spreadsheet.

5. **Adding more data**

   a. Students access the CMS dilepton data set from the link below. There are separate tabs for muons and electrons. The same equations apply for Z-boson decay via muons and via electrons. <u>Gentle reminder</u>: The value of E1 (GeV) is equivalent to the magnitude of p1 (GeV/c) and the value of E2 (GeV) is equivalent to the magnitude of p2 (GeV/c). This practice works for the low mass, high momentum nature of these particles.
   Z Activity dilepton CMS student_data
   https://docs.google.com/spreadsheets/d/1CEg7DIcQ4OW_ETC1v70T3tMR8o42VQzHml5pL0BRpxc/edit#gid=98967810

   b. There are two sheets: *CMS dimuon data* and *CMS dielectron data*. You can divide the class into two groups and have each group copy over the data from one of the sheets. This allows for a discussion of the differences in the results and what that means for the uncertainty in the Z mass calculation. The results are shown below.





<u>**Student Questions**</u>

These questions are intended as formative assessment during class discussion.
Sample answers are reported in *italics.*

1. What is the best value for the Z mass using eight events? Estimate the uncertainty in your answer.
    - *Recall that the mean mass for the Z boson is best represented by the value of the peak of the data. The value should be approximately 90 GeV/c^2 If the peak has width, the students should estimate the uncertainty in their value.*
2. Describe how the histogram changed when more data was used.
    - *When more data is used the peak should become more defined with a clear central peak.*
3. Did the uncertainty in your answer increase or decrease when more data is used?
    - *As the histogram of a normal distribution gets taller, the full width at half maximum tends to decrease,*

Optional Extension:
Since both the dimuon and dielectron experiments are attempting to determine the Z boson mass, another extension is to combine all of the data. This histogram can lead to interesting discussions about histogram uncertainty.