WHAT HEISENBERG KNEW
TEACHER NOTES

DESCRIPTION

Werner Heisenberg proposed the uncertainty principle, one of the foundational concepts of quantum physics, in 1927. Heisenberg proposed that there are pairs of complementary variables which are fundamental quantities of nature. For complementary variables, the greater the precision in the measurement of one variable, the less the precision in the measurement of the other variable. This give-and-take is not from experimental systematics but is part of the very nature of the act of measuring the variables. The best known and most important of these complementary pairs are momentum-position and energy-time. This activity takes an empirical approach to these pairs. Students plot measurements of uncertainty in one variable, e.g., momentum (Δp), as a function of uncertainty in the other variable, e.g., position (Δx), and use these plots to discover relationships between the variables.

STANDARDS ADDRESSED

Next Generation Science Standards

- Science and Engineering Practices
  2. Developing and Using Models
  4. Analyzing and Interpreting Data
  5. Using Mathematics and Computational Thinking
  6. Constructing Explanations and Designing Solutions
  7. Engaging in Argument from Evidence
  8. Obtaining, Evaluating, and Communicating Information
- Disciplinary Core Ideas – Physical Science
  - PS1.A: Structure and Properties of Matter
  - PS2.B: Types of Interactions
- Crosscutting Concepts
  1. Patterns
  2. Cause and Effect: Mechanism and Explanation
  3. Scale, Proportion, and Quantity
  4. Systems and System Models

Common Core Literacy Standards

- Reading
  - 9-12.4 Determine the meaning of symbols, key terms . . .
  - 9-12.7 Translate quantitative or technical information . . .

Common Core Mathematics Standards

- MP2. Reason abstractly and quantitatively.

AP Physics 1: Algebra-Based and AP Physics 2: Algebra-Based Science Practices

- Science Practice 4
  - The student can plan and implement data collection strategies in relation to a particular scientific question.
- Science Practice 5
  - The student can perform data analysis and evaluation of evidence.

IB Physics

- Topic 1: Measurement and Uncertainties
  - 1.2.6 Describe and give examples of random and systematic errors.

1.2.7 Distinguish between precision and accuracy.
1.2.8 Explain how the effects of random errors may be reduced.
1.2.11 Determine the uncertainties in results.
Topic 12: Quantum and Nuclear Physics
12.1: The interaction of matter with radiation

Enduring Understanding

Scientists must account for uncertainty in measurements when reporting results.

Learning Objectives

Students will know and be able to:

- Make plots of data showing uncertainty in the complementary variables.
- Manipulate data to create straight-line plots and thus create a mathematical model of the relationship between complementary variables.
- Explain the uncertainty principle from empirical evidence.

Prior Knowledge

Students should be able to:

- Graph from a table.
- Manipulate data to "linearize" a graph.
- Describe a diffraction pattern.

Background Material

Werner Heisenberg (1901–1976) was one of the most important physicists in the formation of quantum mechanics. In 1927, he proposed the uncertainty principle. It stated that pairs of complementary variables in physics had minimal measurement uncertainties based on a relationship with each other: less uncertainty in one inevitably yields greater uncertainty in the other, no matter how sophisticated the measurement technique.

One way to explain the complementary nature of momentum and position is in terms of wave-particle duality. Imagine that we want to measure the momentum and position of a moving particle that we will call the "target." To do this, we fire a "projectile" with some momentum of its own at the target particle. If the momentum of the projectile is small, it will have only a small effect on the momentum of the target. See Figure 1 below.

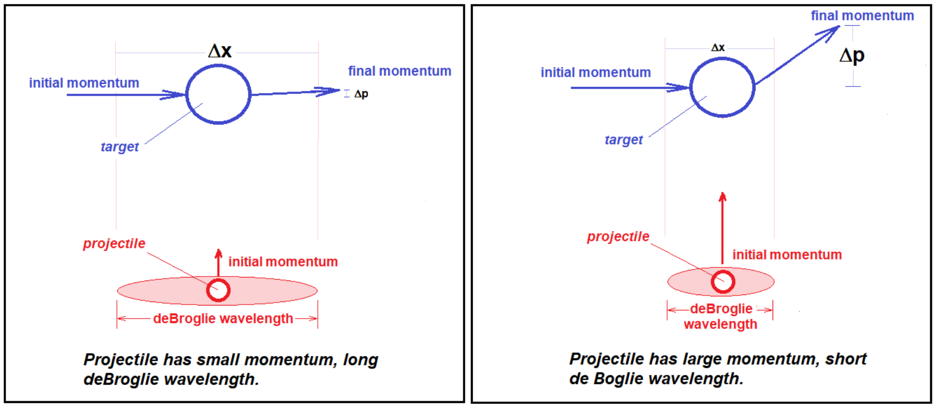


Figure 1: Relationship between momentum and de Broglie wavelength.

When the projectile has a low momentum, as shown in the left picture of Figure 1, the momentum of the target is changed by only a small amount. The projectile bounces back to our detector and its recoil gives us a good idea of the momentum of the target. But, with a low momentum, the projectile has a large de Broglie wavelength. Thus, any measurement of position the projectile makes has a high uncertainty. To improve the position measurement, we can decrease the de Broglie wavelength only by increasing the momentum of the projectile, as shown in the right picture of Figure 1. But if the projectile momentum is greater, then the projectile has a greater effect on the momentum of the target particle, making the momentum measurement less precise. Thus, "you can't win" at a fundamental level. Or at least you cannot totally win: the uncertainty principle quantifies the closest you can come to winning for complementary variables.

In this activity, students discover this relationship from empirical data on momentum uncertainty and position uncertainty for hot fullerene molecules passing through a series of slits of variable width. Done by Olaf Nairz, Markus Arndt, and Anton Zeilinger in 2001, this experiment confirmed the uncertainty principle. As shown in Figure 2 below, the molecules passed through narrow slits of variable width (Δx). Because the molecules were quantum objects, their de Broglie wavelengths caused diffraction, meaning that individual molecules would have seemingly random individual paths after passing through the slit which would, statistically, match a diffraction pattern.

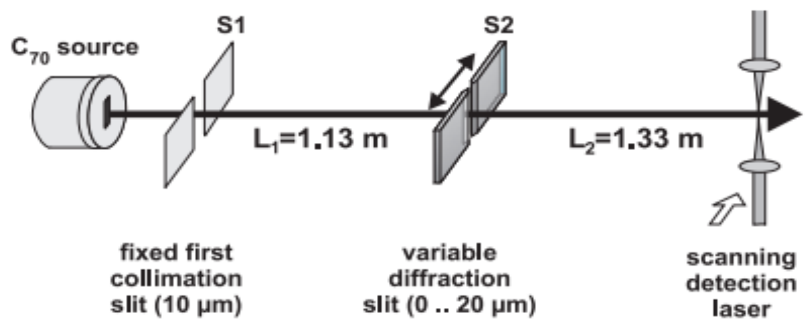


Figure 2: *Experimental setup made by Nairz, Arndt, and Zeilinger, 2001,* *https://arxiv.org/abs/quant-ph/0105061*.

Measurement of the slit width yields the uncertainty in position. Measurement of the width, or angular spread, of the central maximum of this pattern yields the uncertainty in momentum (Δp). Their results were plotted in Figure 3 below. Your students have a table of data taken from the plot.

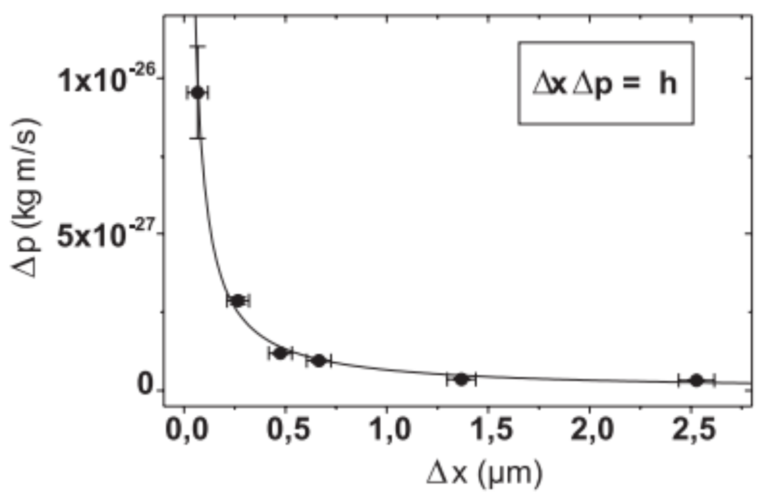


Figure 3: *Results of experiment by Nairz, Arndt, and Zeilinger, from their paper,* *https://arxiv.org/abs/quant-ph/0105061*.

Your students reproduce the plot but also make a straight-line plot of Δp vs. 1/Δx to establish the relationship $\Delta p \propto 1/\Delta x$ or $\Delta p\Delta x$ = constant. In reality, ΔpΔx is greater than or equal to a quantity related to Planck's constant. Finding the value of the proportionality constant is not a goal of this activity.

RESOURCES/MATERIALS

The links below are useful resources:

- Georgia State University HyperPhysics, *Particle lifetimes from the uncertainty principle*, http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/parlif.html
- Olaf Nairz, Markus Arndt, and Anton Zeilinger, *Experimental verification of the Heisenberg uncertainty principle for fullerene molecules*, *Phys. Rev.* A 65, 032109, 5 February 2002, https://arxiv.org/abs/quant-ph/0105061
- Particle Data Group, *Review of Particle Physics*, http://pdg.lbl.gov/
- Wikipedia, *Uncertainty principle*, https://en.wikipedia.org/wiki/Uncertainty_principle
- Wikipedia, *Werner Heisenberg*, https://en.wikipedia.org/wiki/Werner_Heisenberg

Data tables

Materials for making a graph or software for graphical analysis

IMPLEMENTATION

Divide your students into groups of 2–3. Give each group the student pages.

Part 1:

Data Table A has the hot fullerene data for Δp and Δx. Advise your students to plot Δp on the vertical axis and Δx on the horizontal axis as shown in Figure 4.

Data Table A: Complementary Variables Momentum (p) and Position (x)

Uncertainty in Position, Δx (micrometers)	**Uncertainty in Momentum, Δp ($\times 10^{-27}$ kg-m/s)**	**Reciprocal 1/Δx (1/ µm)**
0.09	9.6	
0.28	2.8	
0.46	1.3	
0.65	1.0	
1.36	0.5	
2.52	0.3	

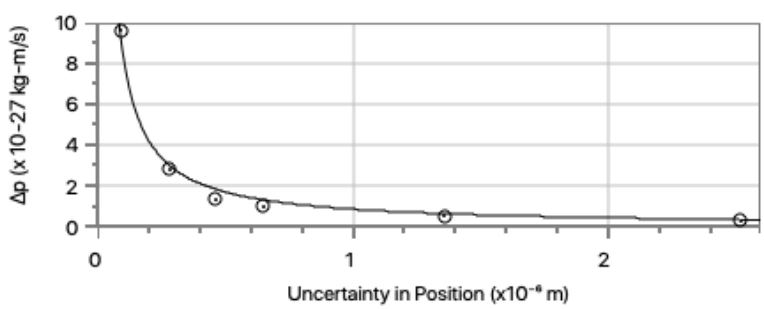


Figure 4: Plot of Δp vs. Δx.

When linearizing data for an inverse graph, the inverted variable can be on the vertical or horizontal axis. Plotting Δp and 1/Δx allows units for the slope that are easier to interpret. See Figure 5.

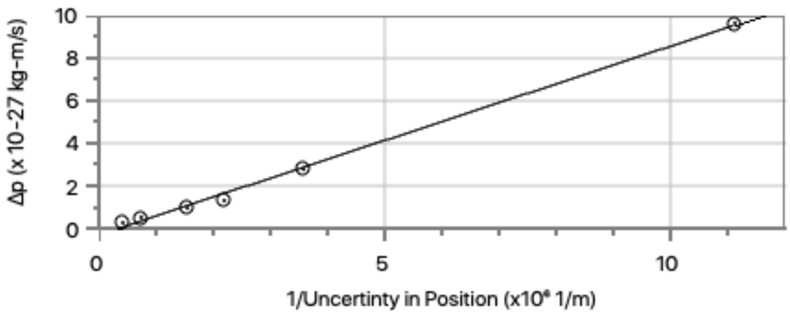


Figure 5: Linearized plot of Δp vs. 1/Δx.

The mathematical model for this linearized data follows:

$$\Delta p = slope * \frac{1}{\Delta x}$$
$$\Delta p \Delta x = slope$$

ASSESSMENT

You can assess this activity using formative assessment in which each group makes a whiteboard presentation of their graphs and makes claims about how well the data supports the claim that Δp and Δx are complementary variables. Another approach is a class discussion.

For summative assessment, you can use the data provided in Table B from the Particle Data Group *Review of Particle Physics* (PDG) to compare the widths of mass plot resonances of selected mesons with the lifetimes of the mesons. Since mass has an energy equivalent, the resonance width is a stand-in for uncertainty in energy, ΔE, and the lifetime for uncertainty in time, Δt. **Note:** Some of the meson "data" is simulated to fill out the data table. Students analyze the data in the same way they do the hot fullerene data but this time for ΔE and Δt.

<u>Research Question</u>:

Are energy and time complementary variables?

Data Table B has the lifetime mass plot data for ΔE and Δt.

Data Table B: Complementary Variables Energy (E) and Time (t)

	Uncertainty in	**Uncertainty in**	**Reciprocal**
	Energy, ΔE	**Time (lifetime), Δt**	**1/Δt**
Meson Name	**(keV)**	**(x 10^{-24} s)**	**(x 10^{24}/s)**
sim1	20	33000	
sim2	40	16000	
upsilon	54	13000	
J/Psi	93	8000	
sim3	135	4900	
f-prime	196	3360	

Source for actual mesons upsilon, J/Psi, and f-prime: Particle Data Group, *Review of Particle Physics*, http://pdg.lbl.gov/

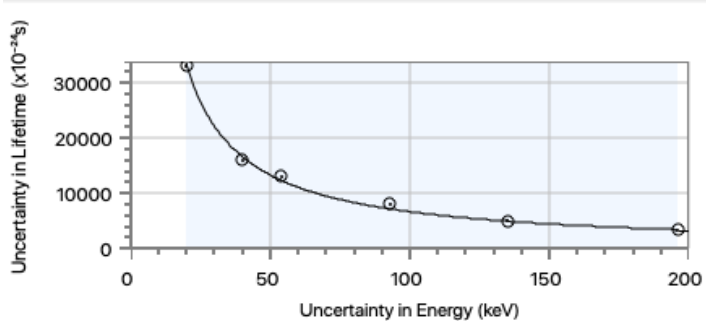
Source for sim calculations: Georgia State University HyperPhysics, *Particle lifetimes from the uncertainty principle*, http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/parlif.html

Student Instructions:

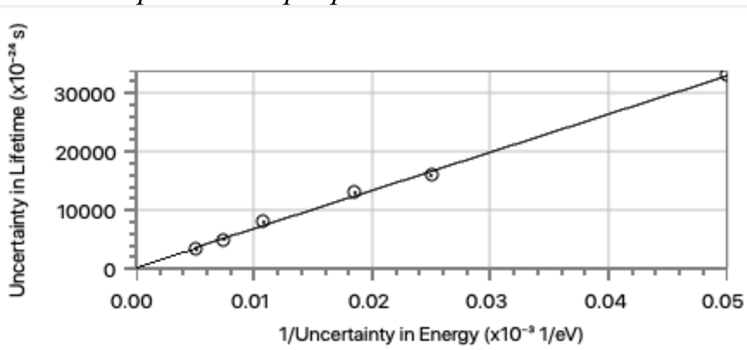
Make a claim about whether energy and time are complementary variables. Justify your claim with evidence and reasoning.

Assessment Scoring:

- **Plot** ΔE vs Δt.



- **Describe** the shape of the graph.
  - *The shape of the Δt vs. ΔE graph shows an inverse relationship.*
- **Make a claim** about what happens to Δt when ΔE increases.
  - *As ΔE increases, Δt decreases.*
- **Determine** the necessary steps to linearize the graph.
  - *When linearizing the data for an inverse graph, the inverted variable can either be on the vertical or the horizontal axis. Plotting Δt and 1/ΔE allows for units for the slope that are easier to interpret. A sample plot is shown below.*



- **Determine** the mathematical model described by the linearized graph.
  - The mathematical model for this linearized data follows:

$$\Delta t = slope * \frac{1}{\Delta E}$$
$$\Delta t \Delta E = slope$$

- **Validity** of claims, evidence and reasoning:
  - *The student cites the shape of the graph Δt vs. ΔE as evidence of an inverse relationship.*
  - *The student correctly determines the equation of the Δt and 1/ΔE as shown above.*
  - *The student makes the claim that Δt and ΔE are complementary variables.*

- *The student concludes that the Heisenberg uncertainty principle applies to the complementary variables Δt and ΔE.*