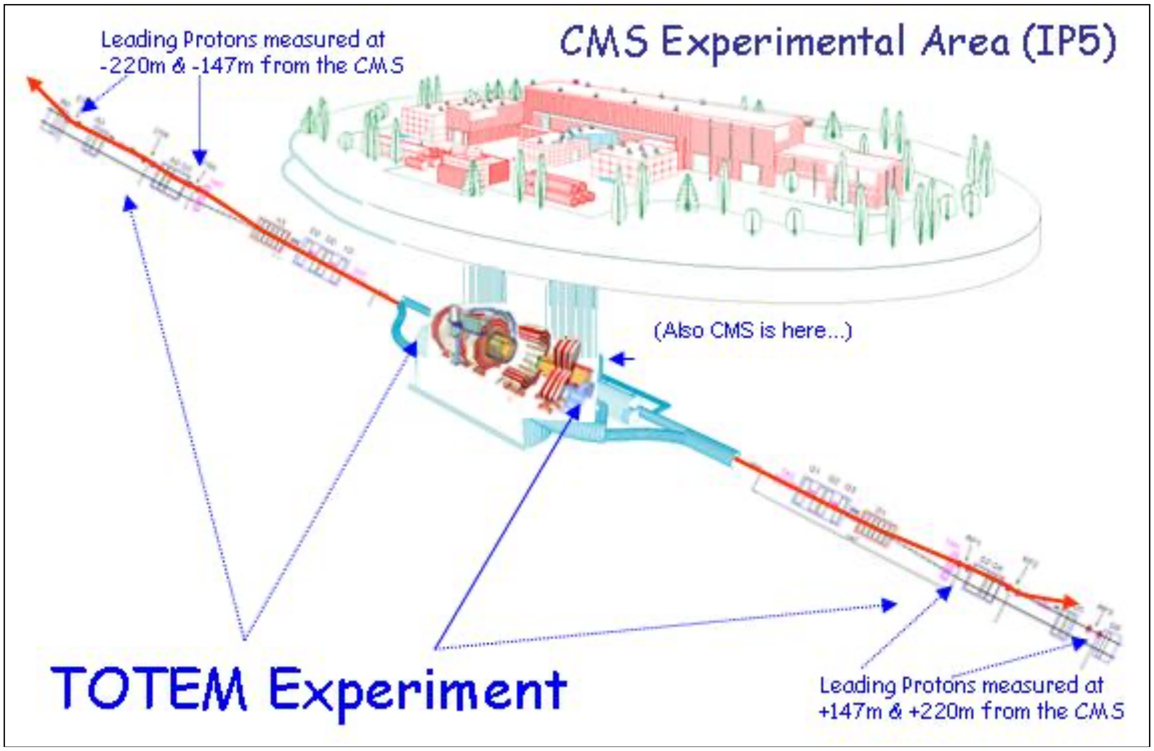
TOTEM I: MOMENTUM
TEACHER NOTES

DESCRIPTION

The *TOTal cross section, Elastic scattering and diffraction dissociation Measurement* (TOTEM) experiment is designed to understand the elastic collisions of protons in the Large Hadron Collider (LHC) at CERN. As stated on the [TOTEM website](), "TOTEM's physics program is dedicated to the precise measurement of the proton-proton interaction cross section, as well as to the in-depth study of the proton structure which is still poorly understood." TOTEM detectors are installed just adjacent to the LHC beamline far forward (220 m) on either side of the Compact Muon Solenoid (CMS) detector. While CMS looks at the results of elastic and inelastic scattering of quarks and gluons in near or completely head-on collisions of protons of which they are components, TOTEM looks at the results of more glancing collisions from which the protons emerge intact. The scattering of these protons is at extremely shallow angles, generally on the order of one ten-thousandth of a radian or a thousandth of a degree.



https://www.lhc-closer.es/taking_a_closer_look_at_lhc/0.totem

The results of this experiment give students and teachers the opportunity to investigate what happens when quantum objects like protons non-destructively interact. These investigations are presented in three parts which can each stand alone or can work together as a unified whole. In this first part, students investigate whether these collisions of quantum objects obey conservation of momentum as do classical particles. The students examine the x- and y-components of the scattering angles of individual pairs of colliding protons and combine their data from many such collisions to see if colliding protons emerge with equal and opposite paths.

STANDARDS ADDRESSED

Next Generation Science Standards

Science and Engineering Practices

4. Analyzing and interpreting data
5. Using mathematics and analytical thinking

8. Obtaining, evaluating and communicating information

Crosscutting Concepts

1. Patterns.
3. Scale, proportion, and quantity.
4. Systems and system models.

Common Core Literacy Standards

Reading

9-12.4 Determine the meaning of symbols, key terms . . .

9-12.7 Translate quantitative or technical information . . .

Common Core Mathematics Standards

MP1. Make sense of problems and persevere in solving them.

MP2. Reason abstractly and quantitatively.

MP4. Model with mathematics.

ENDURING UNDERSTANDING

- Particle physicists use data to determine conservation rules.

LEARNING OBJECTIVES

Students will know and be able to:

1. Make observations by examining a data table.
2. Create and interpret scatter plots generated from the data.
3. Interpret the slope and intercept of the scatter plots.
4. Use measurements to show that quantum particles (protons) conserve momentum in their interactions.

PRIOR KNOWLEDGE

Students must be able to:

- Plot a straight line from data and determine values for the slope and intercept.
- Make connections between momentum and energy in classical mechanics.

BACKGROUND MATERIAL

These resources on TOTEM, the LHC and the physics related to the activity are included on the student pages:

- LHC: The Large Hadron Collider video: https://www.youtube.com/watch?v=debQ60QVtYQ.
- TOTEM on their website: http://totem-experiment.web.cern.ch/totem-experiment/ and in the video: https://www.youtube.com/watch?v=YsZhwu32Zaw.
- Classical elastic and inelastic collisions: https://www.youtube.com/watch?v=M2xnGcaaAi4

RESOURCES.

Data file: https://web.quarknet.org/files/totem/totem1_events_sm_22sep2022.pdf

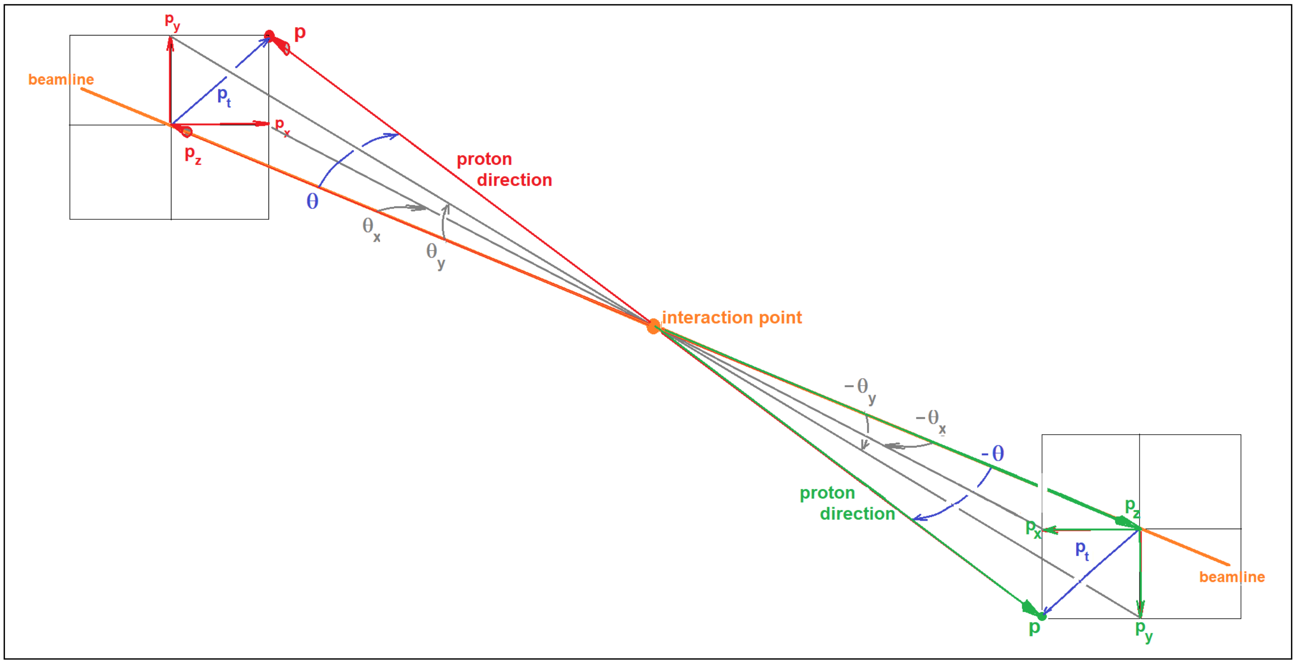
Tally sheet: https://web.quarknet.org/files/totem/totem1tally_20sep2021.pdf

There are two data points on each event slide represented by a red star and a green X. The red stars represent protons detected by TOTEM in one direction forward of CMS and green Xs represent protons detected in the opposite forward direction. Instructions for finding the ordered pairs for each data point are included in the first slide in the data file.

PHYSICS AND GEOMETRY DISCUSSION

This section describes the relationship among the various angles in this experiment. It is important that you, the teacher, understand these relationships, but it is essential that the students discover the relationships from the data.

The angles in the data events are directly related to the momentum vector. This relationship is explained in the figure below:



The momentum vectors for the two protons after the collision are shown as the red and green arrows from the interaction point, where the two protons collide, to the red stars and green Xs on the blue double circle seen in the event display (but not shown here). The momentum vector ***p*** *makes an angle* θ *with the beamline. The components of momentum,* $\mathbf{p_x}$ *and* $\mathbf{p_y}$*, make angles* θ_x *and* θ_y*, respectively, with the beamline. This image must be viewed in color.*

Before the protons (labeled in red and green) collide, they move completely on the z-axis (beamline) but in opposite directions. Since these identical particles are accelerated by the same accelerator at the same energy, the protons must have equal and opposite momenta (4 TeV/c). Thus, the net momentum of the system before the collision is zero. The collision is elastic, so the two protons remain intact. If momentum is conserved, then the net momentum after the collision is also zero. Assume the "red" proton travels in the +z direction. The total momentum of the system before the collision is described by:

$\mathbf{p}_{0(red)} + \mathbf{p}_{0(grn)} = (4\text{ TeV/c})\mathbf{k} + (–4\text{ TeV/c})\mathbf{k} = 0.$

The protons collide and move off with momentum in 3 dimensions. By conservation of momentum,

$\mathbf{p}_{red} + \mathbf{p}_{grn} = 0$ after the collision.

Thus

$\mathbf{p}_{red} = -\mathbf{p}_{grn}$

and, therefore,

$p_{x(red)}\mathbf{i} + p_{y(red)}\mathbf{j} + p_{z(red)}\mathbf{k} = -p_{x(grn)}\mathbf{i} - p_{y(grn)}\mathbf{j} - p_{z(grn)}\mathbf{k}.$

Looking at components individually,

$p_{x(red)} = -p_{x(grn)}$, $p_{y(red)} = -p_{y(grn)}$, and $p_{z(red)} = -p_{z(grn)}$.

We can confirm this in the diagram and, by inspection, see that:

- The right triangle formed by $p_{x(red)}$, $p_{z(red)}$, and the interaction point makes angle θ_x between $p_{z(red)}$ and the gray hypotenuse while the right triangle formed by $p_{x(grn)}$, $p_{z(grn)}$, and the interaction point makes angle $-\theta_x$ between $p_{z(grn)}$ and the gray hypotenuse.
- Equally, the right triangle formed by $p_{y(red)}$, $p_{z(red)}$, and the interaction point makes angle θ_y between $p_{z(red)}$ and the gray hypotenuse while the right triangle formed by $p_{y(grn)}$, $p_{z(grn)}$, and the interaction point makes angle $-\theta_y$ between $p_{z(grn)}$ and the gray hypotenuse.

Thus, we can say that

$\theta_{x(red)} = -\theta_{x(grn)}$ and $\theta_{y(red)} = -\theta_{y(grn)}$.

We expect to see this in our results.

IMPLEMENTATION

Implementation should start with a discussion. You can use the above resources to explain the TOTEM experiment and then ask students:

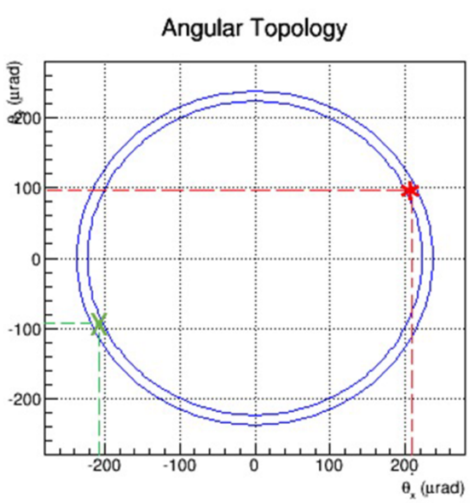
- Is there a relationship between the directions of the two protons after the elastic collision?
- Can you explain why this relationship might or might not exist?

Provide a method for the students to make two scatter plots from -240 μrad to +240 μrad on both axes (see tally sheet). The first graph will have marked axes for θ_x-red vs θ_x-green and the second graph will have marked axes for θ_y-red vs θ_y-green. Instructions for finding the ordered pairs for each data point are included in the first slide in the data file.

The data file contains 21 pages of events, 2 events per page. Students may work in pairs to analyze six or more events. The data file is in pdf format so you can print the events or students can view them on a computer. Plan to oversample the data which means that every event will be analyzed by more than one student team.

Students analyze events by locating the red star and the green X in each event. Each dot represents a detected scattered proton.

Each dot has an x-component, θ_x, and a y-component, θ_y. The students will read off the ordered pair for each dot and enter the values in the tally sheet. The red stars represent proton hits in TOTEM 220 m from the interaction point in one forward direction while the green Xs represent hits from the protons with which they collided on the opposite side, also 220 m away.



The event display shows a planar view, with the beam occupying an area in the middle; there is a gap in the detector there. The TOTEM detectors are embedded at the edges of the LHC beam pipe. Each records a "hit" where a scattered proton strikes. The red stars and green Xs represent these hits.

This sample event shows the main features. We can measure θ_x and θ_y for each red star or green X found between the two blue rings by dropping perpendiculars to the θ_x and θ_y axes. Notice the data points are located between the blue rings. Here, θ_x = 209 μrad and θ_y = 99μrad for the red star and θ_x = -209 μrad and θ_y = 92 μrad for the green X. Details of each event will vary.

Students may use the tally sheet provided in the activity or a tally spreadsheet you or your students develop. Here is an example of what students record in the tally sheet:

Table 1: θ_x

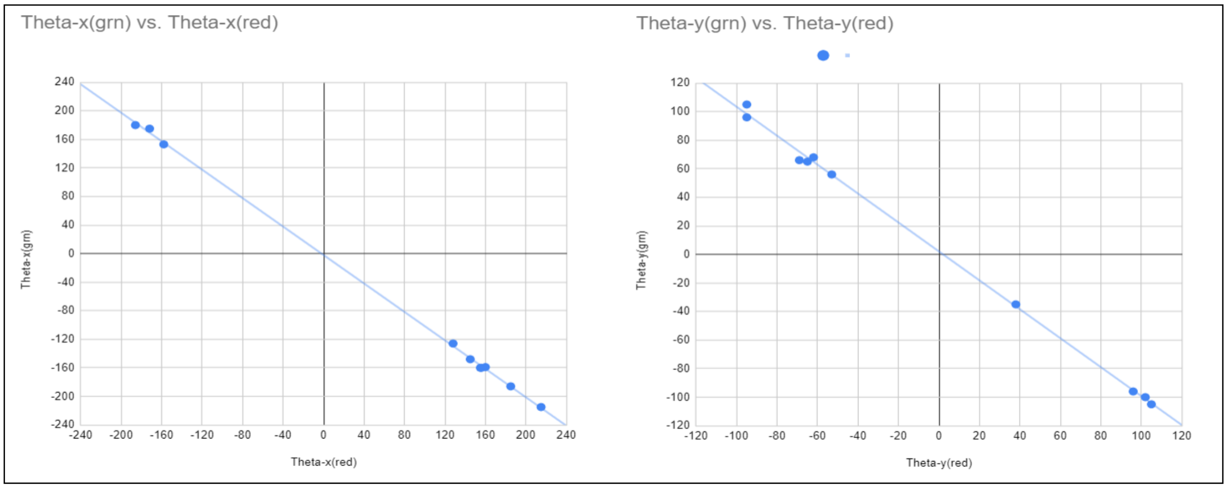
Event Number	θ_x (μrad, red star)	θ_x (μrad, green X)	Δθ (μrad, estimate)
89	+160	-160	5
92	+130	-30	5

Table 2: θy

Event Number	θ_y (μrad, red star)	θ_y (μrad, green X)	Δθ (μrad, estimate)
89	+95	-96	5
92	-88	+89	5

Consolidate the data from all groups into a class set. Here are two possible methods of finding patterns in the data:

1. Inspect their own results, looking at the red and green readings for θ_x and then for θ_y side-by side. They should see that the numbers are almost the same but of opposite sign. Where red and green vary significantly, they should check their result again, taking care to have the correct numbers with the correct signs.
2. Students use their results to create two scatter plots for the whole class. One way to do this is to use "sticky notes" to plot $\theta_{x(red)}$ vs. $\theta_{x(green)}$ and $\theta_{y(red)}$ vs. $\theta_{y(green)}$ on a prepared whiteboard. If they are using a shared spreadsheet, such as the type provided by Google sheets, they can make the plots on the sheet. In any case, students should look for a straight best fit line and find the slope and y-intercept. Once plots are complete, ask the students if they notice any patterns. They should look something like these sample plots below. The sample plots were made from only ten events. A combination class plot should have much more data



In these plots of green Xs vs red stars for θ_x and θ_y, respectively, note that the slopes are very close to -1 and that the vertical axis intercepts are very close to zero. The results, then, are very close to $\theta_{x(red)} = -\theta_{x(grn)}$ and $\theta_{y(red)} = -\theta_{y(grn)}$. Also, note the gaps in the middle of the plots. The gaps show the region that is blocked by the beam pipe, which runs through the center of the detector.

If the protons undergo an elastic collision, the momentum of the protons after the collision should be back-to-back.

Questions for class discussion:

- Examine the data in your tally sheet for θ_x and θ_y. Describe any trends.
  The red star angle is always equal to the negative of the green star value.
- Examine the class plots. What is the value for the slope of the line on each graph? What is the value of the y-intercept of the line on each graph?
  The slope equals -1. The y-intercept is zero.
- Write the equation for each graph.
  $\theta_{x(red)} = (-1)\theta_{x(grn)}$ and $\theta_{y(red)} = (-1)\theta_{y(grn)}$
- Describe the evidence to support or fail to support the claim that quantum objects follow the principles of conservation of momentum.
  The equations indicate that protons, quantum particles, with equal mass collide and go out from the collision in equal and opposite directions.

ASSESSMENT

Assessment is based on accuracy of results as well as claims, evidence, and reasoning. Students complete a report in which they assess the accuracy of the claims and how useful the results are based on evidence and reasoning.

Claims the students will be able to make based on the evidence they build:

- The values for θ_x and θ_y in the tally sheets are the same except for a negative sign.
- The plots provide evidence that quantum objects such as protons follow the law of conservation of momentum.