The 'Nut-Drop' Experiment—Bringing Millikan's Challenge to Introductory Students

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One of the difficulties in teaching 20th-century physics ideas in introductory physics is that many seminal experiments that are discussed in textbooks are difficult or expensive for students to access experimentally. In this paper, we discuss an analogous exercise to Millikan's oil-drop experiment that lets students experience some of the physics involved in the experiment and some of the difficulties Millikan faced.

Millikan's Oil-Drop Experiment

In Robert Millikan's well-known experiment to determine the charge on the electron, small oil drops (or polystyrene spheres in some modern versions) are introduced between two parallel electrodes.[1-3] Viewed through a microscope, the pinpoint specks of light marking the location of the droplets can be seen falling downward under the influence of three forces: gravity, a buoyant force, and a friction force. The droplets quickly reach their terminal velocity due to the friction force, and this terminal velocity can be measured with a scale and a stopwatch or a video camera.[4-6]

The application of an appropriate voltage difference across the electrodes produces an electrostatic force on any non-neutral droplets, and those droplets can be made to rise upward. Measurements of the terminal velocity of the drops as they rise and fall allow for a determination of the electric charge on the droplet.

While this experiment can be done by students in a laboratory, it is generally finicky and difficult to do in a short period of time. For larger classes the technique, but not the data collection, can be done as a demonstration.[7,8] A number of computer simulations of the oil-drop experiment have also been created.[9] However, it is advantageous to give students an alternative activity that can highlight the main aspects of the experiment, while still allowing hands-on access.

Simplifying the Experiment

A main difficulty that Millikan faced was not knowing the number of electrons added to or missing from the oil drops. This is a fairly profound and unusual problem in a physics experiment (particularly in a classroom experiment), where the independent variable is usually well-known. In one common simplification of the oil-drop experiment, designed to highlight this aspect of the problem, students are given small sealed bottles containing an unknown number of identical objects. Measuring the masses of the containers, students find that the mass comes in discrete quantities and that the smallest difference between the masses must be the mass of an individual, quantized object.[10-13] The drawback to these simplified versions is that they are fairly far removed from the actual oil-drop experiment—nothing is actually dropping.

The 'Nut-Drop' Experiment

In this experiment, students are asked to determine the mass of a screw nut using a technique similar to Millikan's. The oil drops are replaced by a collection of identical plastic water-tight containers that hold an unknown number of screw nuts ("elec-

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Fig. 1. Picture of one of the containers (the "oil drop") and the steel nuts (the "electrons").

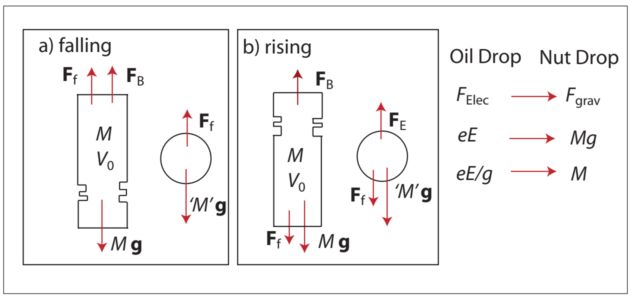

Fig. 2. Schematic showing the forces acting on a) a sinking container and a falling oil drop, and b) a rising container and a rising oil drop. The varying electric force in the oil-drop experiment is compared to the varying gravitational force in the nut-drop experiment, showing how the mass takes the place of the charge of the electron. '*M*' is the effective mass of the oil drop, taking account of the buoyant force.

trons") hidden inside (see Fig. 1). Depending upon the number of nuts inside, a container will either rise or fall when placed in water. Too few nuts correspond to a positively charged oil drop and too many correspond to a negatively charged oil drop. So in this experiment, the buoyant force on the container is constant (taking the place of the force of gravity in Millikan's experiment), and the force of gravity changes (taking the place of the electrostatic force) (see Fig. 2). A number of these containers are allowed to rise or fall through a known distance in a clear tube of water, and a measurement of their terminal velocities can lead to a determination of the mass of an individual nut, even though the number of nuts in the containers is unknown.

The forces on the container:

As the container moves downward in the water, the net force on it is

$$F = Mg - F_f - F_B = Ma, \tag{1}$$

where M is the total mass of the container plus the nuts, g is the acceleration of gravity, F_f is the friction force, F_B is the buoyant force, and a is the acceleration of the container. (Downward acceleration is positive.)

Millikan's oil drops were small spherical particles moving slowly through air (< 0.01 m/s), so they experienced a drag force proportional to their velocity according to Stokes' law.[1] The friction force on a rapidly moving, flat-nosed object should be nearly proportional to the velocity squared:

$$F_f = Cv^2, \tag{2}$$

where C is the drag coefficient for the container.

The buoyant force is equal to the weight of the water displaced by the container:

$$F_B = \rho_w V_0 g, \tag{3}$$

where ρ_w is the density of the water and V_0 is the volume of the container. Once the container reaches its terminal velocity, v_{term}, the acceleration is zero, and we find that

$$v^2_{term} = (Mg - F_B)/C = (M - \rho_w V_0)(g/C). \tag{4}$$

But the total mass of the container and the nuts is

$$M = m_0 + Nm_n, \tag{5}$$

where m_0 is the mass of the container alone, m_n is the mass of a single nut, and N is the number of nuts in the container. So the terminal velocity squared is linearly related to the number of nuts in the container:

$$v^2_{term} = m_n(g/C)N + (m_0 - \rho_w V_0)(g/C). \tag{6}$$

A plot of v^2_{term} versus N should produce a graph with a slope of $m_n(g/C)$. g is certainly known, but C is not, so that quantity must be determined for this container before the experiment can be completed, just

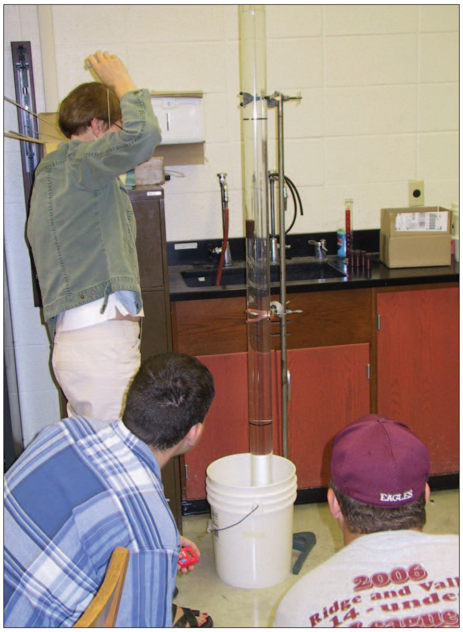

Fig. 3. High school teachers taking data with the apparatus in a modern physics course for secondary school teachers.

as Millikan needed to do. Note that the buoyant force does not play a direct role in the determination of m_n. This is similar to the original experiment, where the buoyant force was folded into the effective mass of the oil drop.

Millikan was able to use Stokes' results[1] to determine the drag force on a sphere in a viscous medium. In the Nut Drop, the drag coefficient must be determined experimentally. Measuring the total mass of a few containers and determining their terminal velocities will lead to a graph of v^2_{term} versus M, whose slope can be used to find g/C from Eq. (4). This measurement could be made earlier in the school year when terminal velocity is introduced. Good documentation of lab work is important, and purposefully using results from months earlier may help convince students it is important and worthwhile.

Determining N:

The remaining problem is to determine N for each of the containers. Luckily, the precise values of N can be replaced with a series of smaller integers, since only the slope of the v^2_{term} versus N line is needed. The data are just sorted into groups that have the same terminal velocity, and then each group can be assigned a value of N, starting with $N = 1$ for the group with the smallest terminal velocity. The v^2_{term}-versus-N plot can then be made and the slope determined. Analyzing the sinking containers separately from the rising containers is helpful.

Experiment Details

The tube used was 3-in (7.6-cm) o.d., 1/4-in (0.64-cm) wall, 6-ft (1.83-m) long transparent plastic pipe with one end closed with a rubber stopper and placed into a five-gallon bucket. A large clamp stand was used to hold the pipe upright. (Clamping the tube to a lab bench is preferable.) The bucket was used to contain the inevitable loss of water. We recommend using a standard-sized transparent PVC pipe and capping the bottom end with a properly affixed PVC cap.[14] Two lines were placed on the pipe one meter apart and at equal distances from the ends of the pipe. The containers must reach their terminal velocity before reaching the first line on the pipe, so the distance from each line to the end of the pipe must not be too short.

A number of containers[15] were filled with varying amounts of steel (not stainless steel) 1/4-in 20 nuts. The containers were then separated into either "sinkers" or "floaters." A rare-Earth magnet from an old hard drive was held next to the wall of the pipe in order to move the container to the starting position and hold it in place. The container was pulled below the water line and shaken to remove any air bubbles attached to it. To release the container, the pipe was held stationary (to keep it from swaying) and the magnet pulled straight outward from the pipe wall. Observers with stopwatches started timing as the container passed the first line and stopped timing when it passed the second line at the 1-m mark (Fig. 3). The magnet was used to pull the container back to the top of the tube. For "floaters," the same procedure was used, except that the container was pulled down to the bottom of the pipe with the magnet before releasing it.

To determine the value of g/C (if not already available from previous experiments), several containers were partially filled and their masses determined with a digital balance. The terminal velocities of these containers were measured the same way, and the terminal velocity squared was plotted versus the mass. The equations of the fit lines from Fig. 4 are:

sinker data:

$$v^2_{\text{term}} = \left(43.4 \pm 3.9 \frac{\text{m}^2}{\text{s}^2 \cdot \text{kg}}\right) \text{mass} + \left(-1.14 \pm 0.14 \frac{\text{m}^2}{\text{s}^2}\right) \quad (7a)$$

floater data:

$$v^2_{\text{term}} = \left(-43.9 \pm 2.1 \frac{\text{m}^2}{\text{s}^2 \cdot \text{kg}}\right) \text{mass} + \left(1.098 \pm 0.040 \frac{\text{m}^2}{\text{s}^2}\right) \quad (7b)$$

The values of g/C are given by the magnitudes of the slopes of these lines.

These quantities are necessary for this experiment and are similar to Millikan's measurement of the viscosity of air in order to calculate the drag on a spherical drop using Stokes' law. While we used containers filled with nuts for these measurements, it may be preferable to use a different material in the containers in order to keep the true nut mass "hidden" from students.

Analysis

For each measurement, the average squared terminal velocity was determined and then binned into a histogram (one histogram for "sinkers" and one for "floaters") as shown in Fig. 5. The distribution of these squared velocities allowed the data to be grouped so that values of N (arbitrary values of N, not the true number of nuts in the container) could be given to each group, and then plots of v^2_{term} versus N were made (Fig. 6).

Results

The equations of the fit lines shown in Fig. 6 are:

sinker data:

$$v^2_{\text{term}} = \left(0.1326 \pm 0.0046 \frac{\text{m}^2}{\text{s}^2}\right) N + \left(-0.123 \pm 0.016 \frac{\text{m}^2}{\text{s}^2}\right) \quad (8a)$$

floater data:

$$v^2_{\text{term}} = \left(0.1443 \pm 0.0022 \frac{\text{m}^2}{\text{s}^2}\right) N + \left(-0.0746 \pm 0.0057 \frac{\text{m}^2}{\text{s}^2}\right) \quad (8b)$$

The values of the mass of a single nut determined from the slopes of the lines in Eq. (8) are: 3.06 ± 0.29 g for the "sinker" data and 3.29 ± 0.17 g for the "floater" data. The actual average mass of the nuts was: 3.037 ± 0.002 g. The results are within 0.08σ and 1.5σ, respectively, of the true value.

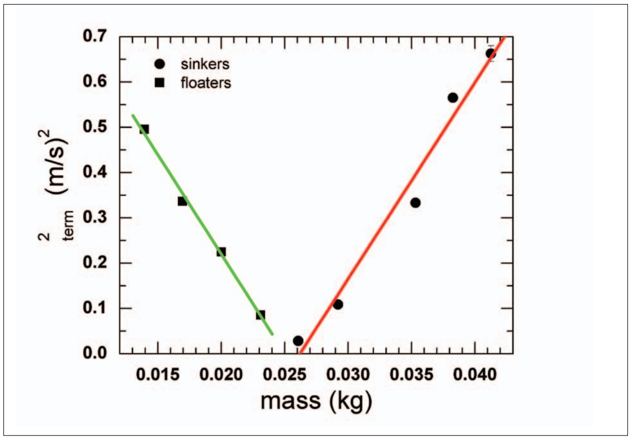

Fig. 4. A graph of the squared terminal velocity vs the measured mass of the container for both the sinking containers (circles) and the floating containers (squares). The error bars are smaller than the symbols for most of the points and represent the standard error of v^2_{term}. Also shown are the linear fits to the data, Eq. (7).

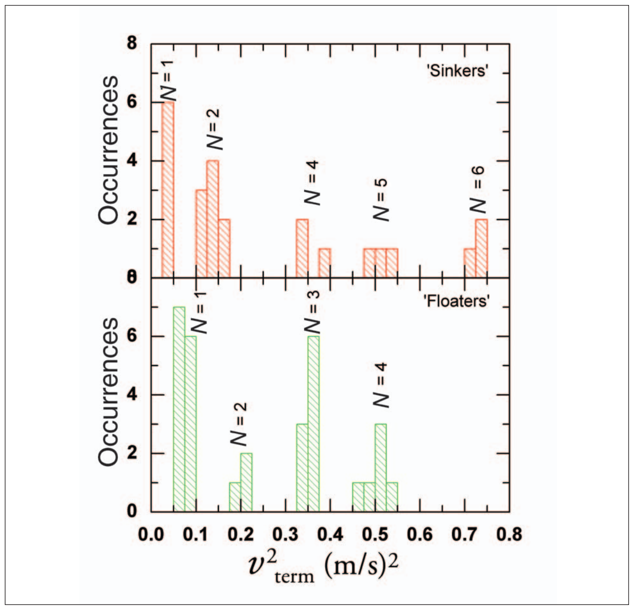

Fig. 5. Histograms of the squared terminal velocities of the "sinker" (top) and "floater" (bottom) containers. The data group themselves according to the apparent number of nuts inside and values of *N* are arbitrarily given to the groups based on the spacing of the groups.

Summary

The results of this experiment turn out fairly well, given the relatively few data points obtained. However, the basic flavor of Millikan's work is retained, and students are left to grapple with the same difficulty that

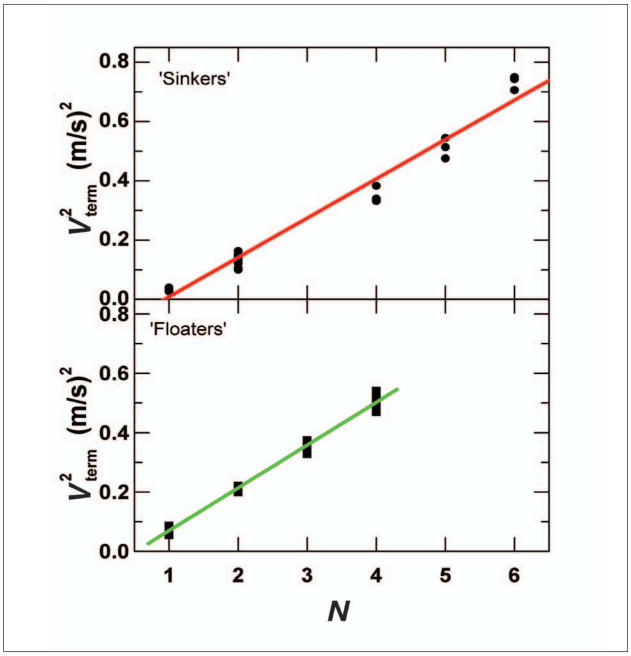

Fig. 6. Graphs of the terminal velocity squared vs the assigned number of nuts based on the histogram in Fig. 5 for the "sinker" (top) and "floater" (bottom) data.

Millikan faced but in a much more accessible, purely mechanical system.

Suggestions

- It is very important to shake off the air bubbles from the container as they can have a noticeable impact on the buoyant force on the container.
- The same end of the container should be used as the "front" end for both rising and falling data, as the two ends have different shapes and will therefore have different values for *C*.
- Make sure that there are multiple containers with the same number of nuts and try to have at least one "missing" number of nuts in the set of containers.
- Measure v_{term} multiple times for a single container, particularly if the container fishtails as it rises or falls. These wandering containers could be viewed as being similar to oil drops in Millikan's experiment that wandered due to Brownian motion.
- Parallax issues are prominent in these measurements, so students need to be encouraged to move up and down to place their eyes level with the starting and stopping lines.
- Using smaller mass nuts as the "electrons" will provide more data points, but then distinguishing different values of *N* becomes much more difficult.

Variations:

- Use only one container and change the number of nuts in it from one trial to the next.
- Determine the mass needed for a container to be approximately neutrally buoyant and add an appropriate amount of sand to all of the containers. This will allow one value of *N* to be a "neutral" droplet.
- Measure m_0 and V_0 and use the intercept from the v^2_{term}-versus-*N* graph to determine g/C, although this requires knowledge of the true number *N*. Alternately, use the convergence point of the v^2_{term}-versus-mass graph (where $F_B = mg$) to find V_0.
- Measure the times using timing gates. This method was used to verify that we allowed adequate distance between the release point and the starting line to reach terminal speed. However, the tendency of the cylinders to drift or "knuckle ball" outside the line of gates introduced some frustration.

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14. One source for such a pipe is McMaster-Carr (www.mcmaster.com), part number 49035K91.

15. Wheaton part number 209124SP, 15-ml amber-colored HDPE narrow-mouth bottle. VWR International catalog number: 16155-124; www.vwr.com.

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